Size Switching in Multiagent Formation Control

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Introduction

- Overview of Multiagent Systems
- Formation Control
- Formation Size Switching
- Simulations
A multiagent system is a team of agents (robots) working in a coordinated fashion to achieve a global objective.

Why multiagent systems?
- Robust
- Scalable
- Inexpensive
Control strategies should be

- Decentralized
- Use only local information
- Robust
- Universal

Global Objective Examples

- Location Rendezvous
- Flocking
- Maintain Formation

Example

Consensus Equation

\[ \dot{x}_i = - \sum_{j \in N_i} (x_i - x_j) \]
Graph Theory

**Graph**

\[ G = (V, E) \]

- **Vertex Set**: \( V = \{1, 2, 3, 4, 5, 6\} \)
- **Edge Set**: \( E \subseteq V \times V = \{(1, 2), (1, 3), ...\} \)
Graph Theory

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  $$= \{(1, 2), (1, 3), \ldots\}$$
Problem Definition
Size-Switching: Narrow Passage

Agents moving in horizontal direction navigating a narrow passage.
Agents moving in horizontal direction navigating a narrow passage.
Size-Switching: Narrow Passage

Agents moving in horizontal direction navigating a narrow passage.
Size-Switching: Obstacle

Agents moving in horizontal direction navigating an obstacle.
Size-Switching: Obstacle

Agents moving in horizontal direction navigating an obstacle.
Size-Switching: Obstacle

Agents moving in horizontal direction navigating an obstacle.
Assumptions

- Each agent $i$ has position $x_i \in \mathbb{R}^d$
- $\dot{x}_i = u_i$ is the control input
- Each agent can measure the relative position and speed of “neighboring” agents
- Each agent can measure clearance $\sigma_i$ and detect the presence of obstacles
Setup

- $N$ robots in $d$-dimensional space
- $x_i \in \mathbb{R}^d$ is state of agent $i$
- Associate graph $G = (V, E)$ with the agents
  - Undirected
  - $(v_i, v_j) \in E$ implies that agent $i$ can measure agent $j$’s relative position and speed
- $G$ is the sensing graph
Defining a Formation

**Formation Specification**

\[
D = \{d_{ij} \in \mathbb{R} \mid d_{ij} > 0, \ i, j = 1, \ldots, N, i \neq j\}
\]

- \(d_{ij}\) is desired distance between agents \(i\) and \(j\)
- The formation must be feasible
- Leads to formation graph \(G_f = (V, E_f, w)\)
  - \(E_f \subseteq V \times V\) is set of edges in the formation
  - \(w\) is edge distance, i.e.
    - \(w : E_f \mapsto \mathbb{R}_+\) and
    - \(w(v_i, v_j) = d_{ij}\)

\(G_f\) not necessarily complete, but rigid
Driving the Agents to Formation

- Define formation potential function \( V : \mathbb{R}^{Nd} \rightarrow [0, \infty) \):

\[
V(x_1, \ldots, x_N) = \frac{1}{2} \sum_{i=1}^{N} \sum_{j \in N_i^f} (||x_i - x_j|| - \alpha_i d_{ij})^2
\]

- \( \alpha_i \) is agent-specific scaling factor
- \( N_i^f = \{v_j \in V : (v_i, v_j) \in E_f\} \)
- \( V(x) \) is positive semidefinite
- \( V(x) \) is only equal to zero when agents agree on \( \alpha \) and have achieved correct interagent spacing
Driving the Agents to Formation

- Establish gradient decent control law

\[ \dot{x}_i = -\frac{\partial V(x)}{\partial x_i} \]

Formation Control Strategy

\[ \dot{x}_i = - \sum_{j \in N^f_i} \left( ||x_i - x_j|| - \alpha_i d_{ij} \right) \frac{x_i - x_j}{||x_i - x_j||} \]

\[ \frac{dV(x)}{dt} = \left( \frac{\partial V(x)}{\partial x} \right)^T \dot{x} = - \left\| \frac{\partial V(x)}{\partial x} \right\|^2 \leq 0 \]
Defining Formation Scale

Discrete number of desired formation sizes

- \( D_{\text{big}} = \alpha_{\text{big}} D \)
- \( D_{\text{small}} = \alpha_{\text{small}} D \)
- \( \alpha_{\text{big}}, \alpha_{\text{small}} \in \mathbb{R}_+ \) and \( \alpha_{\text{big}} > \alpha_{\text{small}} \)

Suppose that if \( j \in N^f_i \), then \( i \) can infer \( \alpha_j \)

Each agent can measure \( \sigma_i \)
Determining Formation Scale

- Define $\bar{\sigma}$ to be threshold clearance ($\bar{\sigma} > \max(D_{\text{big}})$)
- Each agent has continuous scaling factor $\alpha_i \in [\alpha_{\text{small}}, \alpha_{\text{big}}]$
- Establish control strategy for $\alpha_i$

Control of $\alpha_i$

\[ \dot{\alpha}_i = - \sum_{j \in N_i^f} (\alpha_i - \alpha_j) + \gamma_i \]

- $\gamma_i$ is a flow term towards either $\alpha_{\text{small}}$ or $\alpha_{\text{big}}$

\[ \gamma_i = \begin{cases} -c & , \sigma_i < \bar{\sigma} \\ c & , \sigma_i > \bar{\sigma} \end{cases} \]

where $c \in \mathbb{R}_+$

- Saturation limits so that $\alpha_{\text{small}} \leq \alpha_i \leq \alpha_{\text{big}}$
Impose restrictions on sensing graph $\mathcal{G}$:

- If $(v_k, v_i) \in E_f$ and $(v_i, v_e) \in E_f$ then $(v_k, v_i) \in E$ and $(v_k, v_e) \in E$.
- All one-hop and two-hop neighbors of $k$ in $\mathcal{G}_f$ are one-hop neighbors in $\mathcal{G}$.
Observing $\alpha$

$$\psi_i = \sum_{j \in N_i^f} (x_i - x_j) \quad \text{and} \quad \tau_i = \sum_{j \in N_i^f} \frac{x_i - x_j}{||x_i - x_j||} d_{ij}.$$  

**Theorem**

*Given the previous assumptions, $\alpha_i$ is observable from $k$ and*

$$\alpha_i = \frac{e_l^T (\dot{x}_i + \psi_i)}{e_l^T \tau_i}, \quad l = 1, 2, \ldots, d,$$

*where $e_l$, $l = 1, 2, \ldots, d$ are base vectors.*
Narrow Passage
Narrow Passage

www.prism.gatech.edu/~scoogan3/symposium/video1.html
Narrow Passage

\[ \alpha_{\text{big}} = 1 \text{ and } \alpha_{\text{small}} = 0.1 \]
Expansion Around an Obstacle
Expansion Around an Obstacle

(a) (b) (c) (d)
Expansion Around an Obstacle

\[ \alpha_{\text{big}} = 2.5 \quad \text{and} \quad \alpha_{\text{small}} = 1 \]
5 Agents Traversing a World
5 Agents Traversing a World

\[
\alpha_{\text{small}} = 0.2, \quad \alpha_{\text{normal}} = 1, \quad \alpha_{\text{big}} = 3
\]
Interesting Extensions

- Stochastic sensing links
- Dynamic target size adjustment
- Robust analysis of formation size selection
- Interesting property: \[ \sum_i \dot{\alpha}_i = \sum_i \gamma_i \]
  - Can cause formation size to reach static state without coming to consensus on formation size
  - May need to introduce perturbation term so that \[ \sum_i \dot{\alpha}_i \neq 0 \]